Spatial and temporal turbulent velocity and vorticity power spectra from sound scattering.

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By performing sound scattering measurements with a detector array consisting of 62 elements in a flow between two counter-rotating disks we obtain the energy and vorticity power spectra directly in both spatial and temporal domains. Fast accumulated statistics and large signal-to-noise ratio allow to get high quality data rather effectively and to test scaling laws in details.

One of the challenging experimental tasks in studies of turbulent flows is developing new tools to measure spectral characteristics of velocity and vorticity fields in a spatial domain. Such measurements will allow direct comparison of experimental data with a theory without exploiting the Taylor hypothesis particularly in those cases when its use is rather questionable. Currently, from conventional methods in use only particle image velocimetry (PIV) is capable of performing this task on the expense of a very heavy data analysis to get representative statistics and a reasonable signal-to-noise ratio.

In this Letter we present results on direct measurements of the velocity and vorticity fluctuations spatial and temporal spectra by using sound scattering amplitude measurements. We achieve this goal by simultaneous acquisition of sound pulses on 62 sound detectors, arranged in front of a linear emitter in the same plane, as has been already described in Ref. [5]. There were several attempts in the past to experimentally probe turbulent flow characteristics by the sound scattering method. These experiments were mostly concentrated on studies of vorticity distribution [1] as well as temporal dynamics of vorticity fluctuations [2,3] and a large scale circulation in high Reynolds numbers flows [4,5].

As we pointed out [5] the main obstacles to get reliable and complete information about the velocity field from sound scattering are a finite width of a sound beam and far field approximation required by the theory [6–8]. Since an integral scale of the velocity fluctuations is of the order of the beam width, the former limitation is released for the turbulent velocity fluctuations. Thus, to get spatial information about the velocity spectra with sufficient dynamical range from the sound scattering data one needs to exploit a large number of elements in the sound detector array, to use lock-in detection technique to improve signal-to-noise ratio, and to apply the Huygens far-field construction method [5]. Then the amplitude of the complex wave function of the scattered sound in the far-field limit can be related to the Fourier transform of the velocity field (and also the vorticity) in a

scattering plane as follows [7,8]:

$$\Psi_{scat}(\vec{r},t) = \frac{1}{c}\frac{(2\pi k_0)^2 \exp{(i\pi/4)}}{\sqrt{2\pi k_0 r}}\cos\theta F_{\vec{k_s}}\{v_x\}, \quad (1)$$

where $\Psi_{scat} = \Psi - \Psi_{rest}$, Ψ and Ψ_{rest} are the complex wave functions that describe the sound pressure oscillations in the presence of a flow and without it, respectively; k_0 and c are the sound wave number and the velocity, respectively; θ and k_s are the scattering angle and the wave number, respectively, that are related to each other via $k_s \equiv |\vec{k_s}| = 2k_0 \sin\theta/2$; and $r = |\vec{r}|$ is the distance from the center of a scattering region till a detector. $F_{\vec{k_s}}\{v_x\}$ is the 2D Fourier transform of the velocity component in the forward direction of the beam that is related to the Fourier transform of the vorticity via $F_{\vec{k_s}}\{v_x\} = \frac{i}{2k_0}\cot{(\theta/2)}F_{\vec{k_s}}\{(\nabla \times \vec{v})_z\}$.

Since in many cases there exist severe technical obstacles to obtain a signal in a far-field limit, a construction of the far-field scattering wave function from the near-field measurements was suggested and used [5]. The method is based on a mathematical description of the Huygens principle in optics and derived from the Rayleigh-Sommerfeld integral [9] as:

$$\Psi(r_f,y)_{scat}^{ff} = \int \frac{k_0 i^{3/2}dy'}{\sqrt{2\pi k_0(r_f - r_d)}}$$

$$\exp{(\frac{ik_0(y-y')^2}{2(r_f - r_d)})}\Psi(r_d,y')_{scat}^d, \quad (2)$$

where r_d and r_f are the distances measured from the cell center till the detector and the far-field region, respectively; $\Psi(0,y')_{scat}^d$ and $\Psi(x,y)_{scat}^{ff}$ are the scattering wave functions at the detector (as measured) and at the far-field, respectively [5].

The experimental set-up is described in details in Ref. [5]. The von Karman swirling water flow was produced between two counter-rotating disks of a diameter $2R = 280$ mm with four triangular blades of 20 mm high

and 5 mm thick and with rims. They are driven by two dc brushless motors, which velocity is controlled with a stability of about 0.1% via optical encoders. This set-up is well recognized to generate a strong intensity turbulent flow in a confined region (see for example [2]). The flow is confined by a perspex cylinder of an inner diameter 290 mm and 320 mm in height and disks separation 205 mm. By changing a rotation frequency the Reynolds number, $Re = 2\Omega R^2/\nu$, is varied between $2.5 \cdot 10^5$ and $1.7 \cdot 10^6$ that corresponds to the Taylor microscale Reynolds number R_λ between 200 and 570. Here Ω is the angular velocity of the disk, ν is the kinematic viscosity, and the energy dissipation, $\epsilon = 4.9 \cdot 10^{-18} Re^3$ W/kg, and the rms of the velocity fluctuations in the middle plane, $V_{rms} = 0.5 \cdot 10^{-6} Re$ m/sec, obtained from the global torque and the hot wire anemometry (HWA) measurements, respectively [10].

The sound scattering measurements were conducted in the middle plane between the disks and in the plane at 30 mm below it. The emitter was 100 mm long and 10 mm wide, and the size of the scattering region was defined by the width of the detector designed as a linear array of 62 acoustic detectors with 1 mm spacing and 62×10 mm active area (from Blatek). A range of frequencies covered in the experiment was between 1 and 7 Mhz. The acquisition system is built in a heterodyne scheme that is based on 62 lock-in amplifiers with 62 preamplifiers. The details of the design and its operation are presented in Ref. [5]. A typical sound propagation time through the cell is about 200 μsec that is a typical freezing time segment. Within this period one pulse is sent, and the flow is almost frozen. Since an each sound pulse has a sufficiently high signal-to-noise ratio, it is used to construct a sound far-field wave function from sound scattering signals acquired by all detectors. Obtained as a function of time the wave function provides a possibility to study statistics of velocity (vorticity) fluctuations in a turbulent flow.

A scattering wave function emitted from a single transducer and obtained by the detector array provides information about the velocity (vorticity) field only on a single curve in a $((k_s)_x, (k_s)_y)$ plane (see Fig.1, left inset, solid line). To get information on other curves in the plane (see Fig.1, left inset, dashed lines) requires to use sound beams emitted by different transducers in many different directions simultaneously. Then complete structure functions of the velocity (vorticity) fluctuations can be retrieved without any assumption about isotropy and homogeneity of a turbulent flow. However, with only one emitter and the detector array we should rely on the isotropy and homogeneity assumptions of the turbulent flow under studies. In this case the sound scattering provides direct measurements of the energy spectrum as well as the Fourier transform of the vorticity structure function in a spatial domain that are related

as $E(k_s) = \frac{6\pi^3}{Ak_s}|F_{\vec{k_s}}\{(\nabla \times \vec{v})_z\}|^2$. Here the kinetic energy per unit mass is defined as $\int E(k)dk = \frac{3}{2A}\int v_x^2 d^2r$, where A is the cross-section area of a sound beam. We neglect velocity variations in the direction perpendicular to the beam propagation by averaging over the beam thickness. In Fig.1 we present a typical result on the time-averaged velocity Fourier transform in the far-field obtained from the scattering wave function via Eq.(1) and the far-field reconstruction according to Eq.(2). The same function but observed directly in the detector plane looks drastically different (see right inset in Fig.1). These data are the result of averaging on 60,000 sound pulses at 1.8 kHz repetition rate and at frequency 3MHz taken in the von Karman swirling flow at $Re = 1.5 \cdot 10^6$.

In Fig.2 the resulting energy spectrum as a function of the wave number is shown. The dotted line denotes the "-5/3" slope according to the Kolmogorov law to demonstrate that indeed in some range of the wave numbers the spectrum follows the expected law. It can be compared with the results on the energy spectra obtained by PIV, where though a shorter scaling region is also observed [10]. In the sound measurements shown in Fig.2 the wave numbers are limited by the range of values $0.1 < k < 0.8$ mm^{-1}, outside of which the spectrum cannot be retrieved. The lower side of the range of k is limited by the beam width, i.e. one cannot get information on a scale exceeding the beam width (in the energy spectra from PIV the scaling region begins already at $k = 0.06$ mm^{-1} for $Re > 10^6$) . On the higher side of the range the ultimate limit is determined by the size of the element in the detector array, namely $k_{max} \leq 2\pi$ mm^{-1} corresponding to 1 mm detector array spacing. However, even before this limit is reached, the energy spectrum is cut on the higher wave number side by the visibility at large scattering angles through the detector aperture. It means that some sound rays are blocked by the limited length of the detector array. According to the detector array length and the cell diameter at angles exceeding about 6° the visibility starts to deteriorate (see the inset in Fig.2). The corresponding limit is $k_{s(lim)} = 2k_0 \sin(\theta_{max}/2) \simeq 0.1k_0$. It gives about 1.3 mm^{-1} at 3MHz compared to 0.8 m^{-1} observed. We tested this relation at various frequencies, and the results on the energy spectra taken at different sound frequencies are presented in Fig.3. One finds that by changing the frequency from 1 up to 7MHz the upper wave number limit moves linearly toward the highest value of about 1.5 mm^{-1}, but still far away from $k_{max} = 2\pi$ mm^{-1}.

Useful information in terms of the turbulent flow energy dissipation, ϵ, can be gained from combined presentation of the energy spectra obtained from the sound scattering measurements at different Reynolds numbers. There is a known scaling law for the energy spectra [11–13] that appears as the result of plotting the scaled energy density spectrum $\epsilon^{-2/3}\eta^{-5/3}E(k)$ versus

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$k\eta$, where η is the Kolmogorov dissipation scale defined as $\eta = (\nu^3/\epsilon)^{1/4}$ [13]. The idea is to find the best match between the scaled spectra at different Re with fitting values of ϵ. It turns out that the scaling exists for the data at all values of Re, and the results for the sound scattering in the middle plane (curve 1) and in the plane at 30 mm below it (curve 2) are shown in Fig.4. Each data set for each Re consists of $4 \cdot 10^6$ points. The dependence of the energy dissipation, ϵ, on Re is found to be $\epsilon \sim Re^{3.1\pm0.1}$ with a good quality of the fits. The exponent value is rather close to 3, the expected one according to the dimensional analysis [10,14](compare with the expression for ϵ presented above). The Kolmogorov constant C in the Kolmogorov equation $\epsilon^{-2/3}\eta^{-5/3}E(k) = C(k\eta)^{-5/3}$ is determined experimentally from the fit of the plots in Fig.4, and the value is $C \simeq 0.8$ for the curve (1) and $C \simeq 0.9$ for the curve (2).

We also calculate the integral scale in the flow at $Re = 1.2 \cdot 10^6$, using [15] $L_{int} = (3\pi/4) \int k^{-1}E(k)dk / \int E(k)dk = 40 \pm 10$ mm, based on the system scale 0.02 mm^{-1} and the scaling region $0.06 < k < 1.5$ mm^{-1}. This value occurs to be rather close to the beam width, and the corresponding wave number is located close to the lower end of the wave number range of the spectrum.

To test the Taylor hypothesis [13,15] for the swirling flow the energy spectrum in the frequency domain is calculated as $E(f) = \frac{1}{T}\int_0^T E(t)\exp(-i2\pi ft)dt$ and $E(t) = \int \frac{6\pi^3}{Ak_s}|F_{\vec{k_s}}(\nabla \times v(\vec{t}))_z|^2 d^2k_s$. A proper energy spectrum can be obtained only, if the lowest k_s are available due to the pole at k_s^{-1} in the integrand. So we turn out to the Fourier transform of the vertical vorticity. In Fig.5 (upper curve) we present the time-averaged power spectrum of the vorticity as a function of k (based on $2 \cdot 10^7$ points) at $Re = 1.2 \cdot 10^6$ obtained at the sound frequencies 2.5 MHz (asterisks) and 5.8MHz (diamonds). The dashed line with the scaling exponent "-2/3" represents the expected dependence according to the Kolmogorov predictions [13,15]. It can be compared with the space-averaged over the beam area Fourier transform of the enstrophy in the frequency domain $N(f) = \frac{1}{T}\int_0^T N(t)\exp(-i2\pi ft)dt$, where $N(t) = \frac{1}{A}\int_A |(\nabla \times v(\vec{t}))_z|^2 d^2r = (2\pi)^2 \int |F_{\vec{k_s}}(\nabla \times v(\vec{t}))|^2 d^2k_s$. The latter shows a rather wide scaling region (see the lower curve in Fig.5). The advection velocity, V_T, found from the two plots in Fig.5 is $V_T = 2\pi f/k \simeq 0.6$ m/sec that is rather close to the average velocity in this experiment [10].

We would like to point out also that the mean enstrophy value, marked on Fig.5 by asterisk on the left hand-side ordinate axis, is about an order of magnitude larger than one corresponding to a rigid body rotation with the same rotation velocity of 300 rpm and the vorticity of $63\,\mathrm{sec}^{-1}$, or the enstrohy of about $4000\,\mathrm{sec}^{-2}$. In the co-rotational disks geometry at the same parameters and with a large scale single vortex flow configuration the enstrophy is also much lower [5].

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FIG. 1. Time averaged velocity structure function obtained by sound scattering via the far-field construction taken at $Re = 1.5 \cdot 10^6$ and 3 MHz. The right inset: the same property obtained from sound scattering data at the detector without far-field construction. The left inset: path in the wave number plane (solid line),on which information on the velocity (vorticity) spectrum for a given beam direction is obtained; other curves (dashed lines) contain information on scattering from sound beams emitted in various directions.

FIG. 2. Energy spectrum derived from the data presented in Fig.1. The inset: scheme of the aperture limit for the sound detector array.

FIG. 3. (color online) Energy spectra taken at various sound frequencies from 1 up to 7 MHz at $Re = 1.2 \cdot 10^6$.

FIG. 4. (color online) Scaled energy spectra as a function of reduced wave number at various Re: (1) at $h = 0$ (2) at $h = -30$ mm. The dashed lines show the Kolmogorov scaling law with the exponent "-5/3".

FIG. 5. Upper curve: Power spectra of time averaged vorticity as a function of k taken at $Re = 1.2 \cdot 10^6$ and frequencies 2.5 (asterisks) and 5.8 (diamonds) MHz. Lower curve: Power spectra of space-averaged enstrophy as a function of f. The dashed lines show the Kolmogorov scaling with the exponent "-2/3".

Legend:
- 7.0MHz
- 5.5MHz
- 3.5MHz
- 3.0MHz
- 2.5MHz
- 2.0MHz
- 1.5MHz
- 1.0MHz

-5/3

$E(k)$ [mm^3/s^2]

k [mm^{-1}]